MATH 17 – QUIZ 2
20 SEPTEMBER 2010

Name: SOLUTIONS

NO CALCULATORS

Compute the following integrals.

1. $\int x \sin x \, dx$

Use integration by parts.

$u = x \quad dv = \sin x \, dx$

$du = dx \quad v = -\cos x$

$$\int x \sin x \, dx = -x\cos x + \int \cos x \, dx = \boxed{-x\cos x + \sin x + C}$$

2. $\int \frac{1}{\sqrt{1-x^2}} \, dx$

Use trig substitution

$x = \sin\theta \quad dx = \cos\theta \, d\theta, \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}.$

$$\int \frac{1}{\sqrt{\cos^2\theta}} \cdot \cos\theta \, d\theta = \int \frac{\cos\theta}{|\cos\theta|} \, d\theta = \int d\theta = \theta + C = \boxed{\sin^{-1} x + C}$$

3. $\int \frac{1}{x^2+2x+5} \, dx$

Complete the square.

$$\int \frac{1}{(x^2+2x+1)+4} \, dx = \int \frac{1}{(x+1)^2+4} \, dx$$

Let $u = x+1$, then $du = dx$

$$\int \frac{1}{u^2+4} \, du = \int \frac{1}{4\left(\frac{u^2}{4}+1\right)} \, du = \frac{1}{4}\int \frac{1}{\left(\frac{u}{2}\right)^2+1} \, du$$

Let $v = \frac{u}{2}$. $dv = \frac{1}{2} du$

$$\frac{1}{2}\int \frac{1}{v^2+1} \, dv = \frac{1}{2}\tan^{-1} v + C = \boxed{\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C}$$